The Drive to the Aviary

By Lorena Lopez

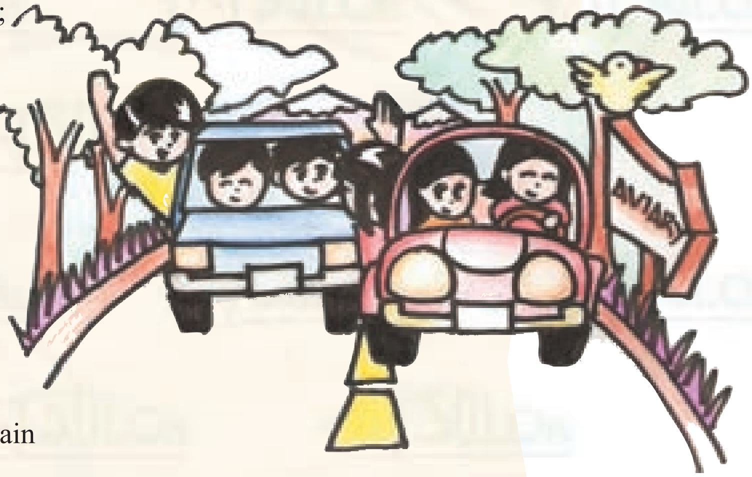
"All set? While we're waiting for Mandy, check out your car assignments," Kat's sister Noems said aloud. "I'll drive our car, with Kat and Minnie; Jonah's brother Tim will drive theirs with Jonah and Geoff; Rei's mom, *Tita* Beth, will have Rei and Lenny; Dan, you will be with Mandy and her driver Mang Fermin when they turn up."

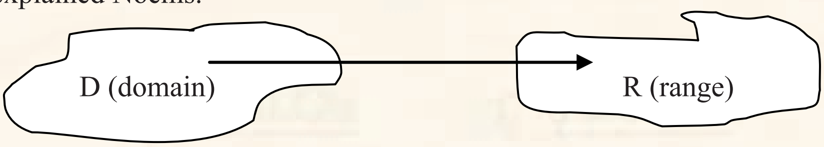
"*Ate* Noems, why can't you just drive all eight of us in the van?" asked Kat.

"It would be difficult for me to function all alone, though you're all grown up," Noems began.

"Oh, function. Now I'm reminded of the confusing one-to-one and onto functions," said Kat.

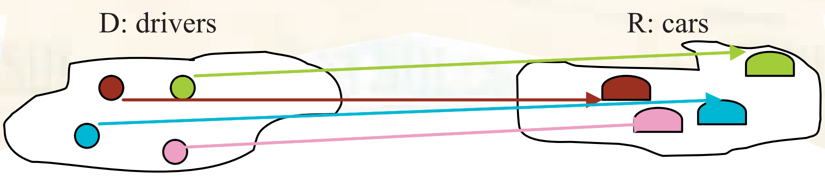
"What about them? Just remember that, first, you are working with two sets: the domain and the range," explained Noems.





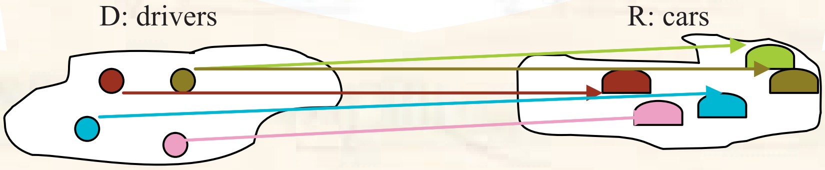
"Yes. And that in a one-to-one function, every value from the domain has exactly one corresponding value in the range. But, how come I can't have an x from the domain that goes to two y's in the range?" asked Kat.

Noems got sheets of paper and colored pens. "Hmmm, well, imagine our trip. Let's have finite sets: the domain is the set of drivers: me, Tim, Tita Beth, and Mang Fermin. Let the range be the cars: ours, Tim's, Rei's, and Mandy's. Is it a one-to-one function?" she asked.



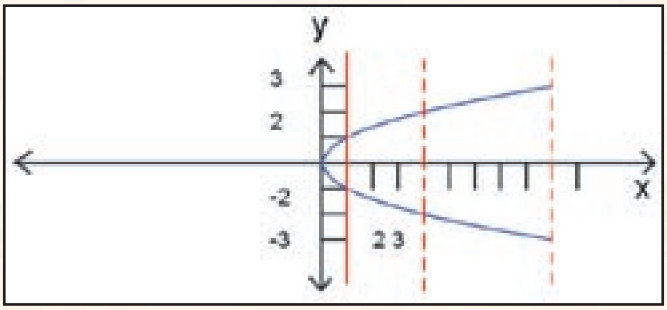
“Of course! For each of you, there’s only one car,” said Kat.

“Exactly, and the inverse is also one to one: each car has exactly one driver. Now, can you imagine me driving two cars?” asked Noems.

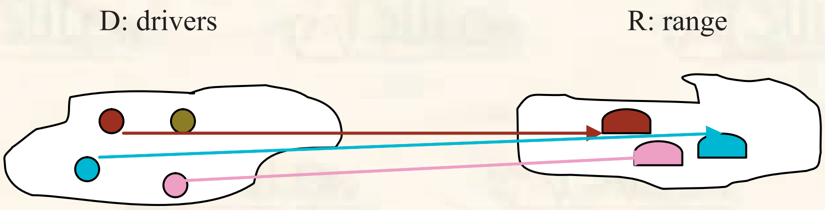


“At the same time? No way! It’s impossible!” reacted Kat.

“Think of that: a function can only have one ending point. Or it would not be a function but just a relation,” Noems said. “Think of $f(x) = \sqrt{x}$. If x were 4, then y could be positive 2, or negative 2. If you sketch its graph and draw a vertical line test, you’ll see that the line would touch more than one point most of the time:



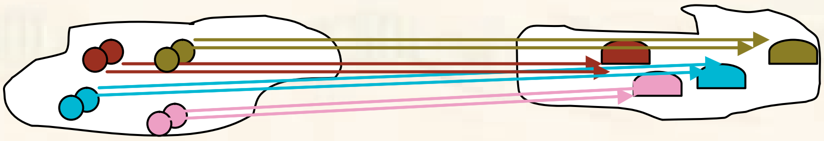
Now, if I don’t have any car to drive, can I perform my function as a driver?”



Kat stayed silent, thinking about the situation. “Umm ... not,” she answered.

The Horizontal Line Test
If the vertical line tests whether a given graph is a function, the horizontal line tests if the function is one-to-one.

"Right," agreed Noems, "a function must have an ending point. Now let's look at the passengers as the domain, and the cars as the range: two can go to a single car. Is that a function?"



"Yes, because all of us has a car to go to," Kat answered confidently. "I can write the set of ordered pairs as {(Kat, car 1), (Minnie, car 1), (Jonah, car 2), (Geoff, car 2), (Rei, car 3), (Lenny, car 3), (Mandy, car 4), (Dan, car 4)}."

"And, that's an onto function. All cars have a passenger," Noems continued. "What if we include the van in the set of vehicles. No passenger goes to it. Would it still be a function?"

"Hmm, it's still a function," Kat said slowly, thoughtfully. "But, is that allowed? It's not one to one, and it's not even onto."

"Yes, you can have a function that's not one to one, and that's not even onto. Go back to our first set of drivers and cars. Is it onto?" Noems asked again.

"Yes. And it's also one to one!" Kat answered with confident.

"Indeed. Such functions are called bijective," agreed Noems.

Kat stood up and said, "So you can have functions that are both, and functions that are neither. Wow, this trip is turning out to be a memorable one for me."

WORKSHEET

1. Determine if the following is one to one, onto, both, neither, or not a function. Show by illustration the domain, the range, and the mapping.

   a. {(1, -1), (2, -2), (3, -3), (4, -4), (5, -5)}

   b. $f(x) = 2x$, where $x \in \{1, 2, 3, \ldots\}$ and $y \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

   c. $f(x) = \frac{1}{3}$, where x can be any number and $y \in \left\{\frac{1}{3}\right\}$

   d. A unit circle on a Cartesian plane with center at point (2, 2) and radius 3

2. Show the graph of each function below and state if it is one to one, onto, both, or neither, with the domain and range as the set of real numbers. Perform the horizontal line test.

   a. $f(x) = 7x + 6$

   b. $f(x) = |x| + 2$

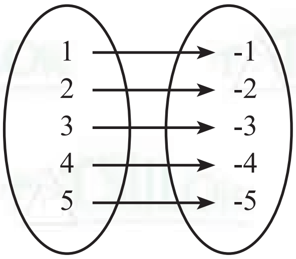
   c. $f(x) = 4x^2 - 1$

   d. $f(x) = x^3 - 5$

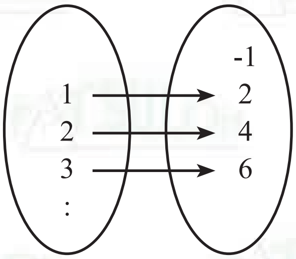
Answer Key:

1. Solution (illustrations may vary):

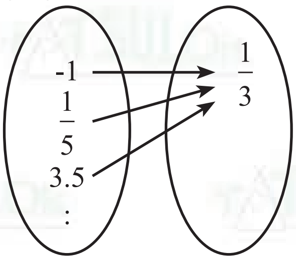
a. D: {1, 2, 3, 4, 5} R: {-1, -2, -3, -4, -5}
both one to one and onto



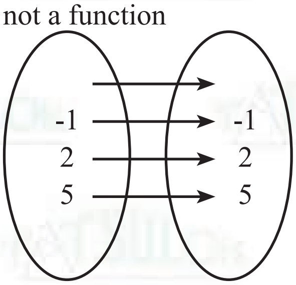
b. D: Positive integers greater than zero (or counting numbers) R: Integers
neither one to one nor onto

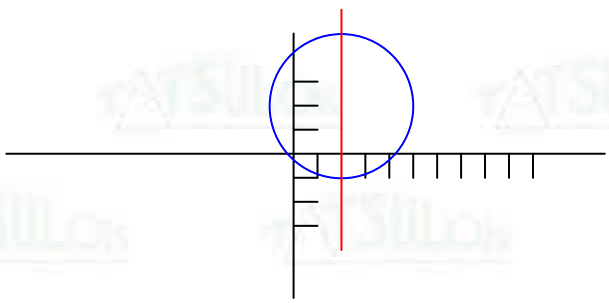


c. D: real numbers R: 1/3
onto



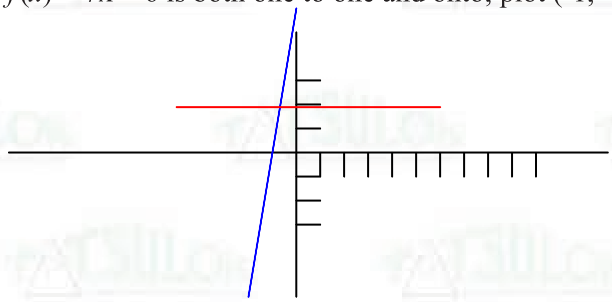
d. D: real numbers R: real numbers
not a function



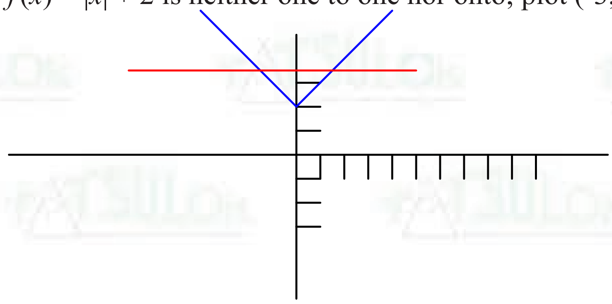


2. Solution (plotted points may vary):

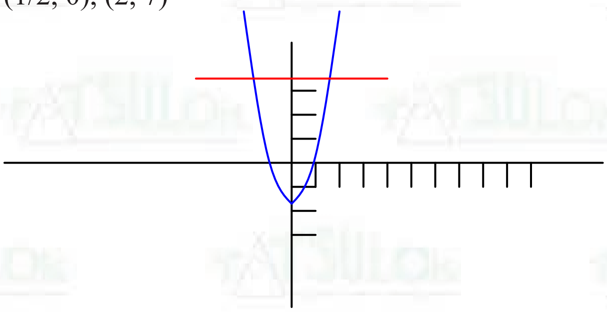
a. $f(x) = 7x + 6$ is both one to one and onto; plot (-1, -1), (0, 6), (1, 13)



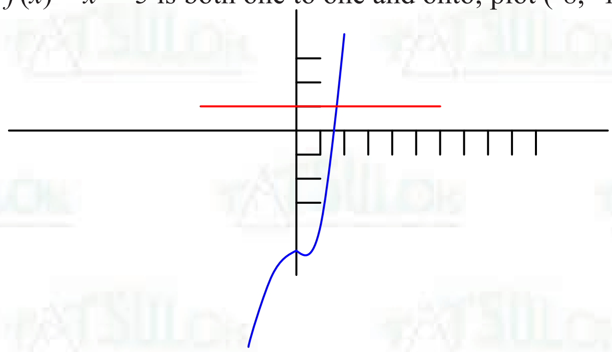
b. $f(x) = |x| + 2$ is neither one to one nor onto; plot (-3, 5), (-1, 3), (0, 2), (1, 3), (4, 6)



c. $f(x) = 4x^2 - 1 = (2x + 1)(2x - 1)$ is neither one to one nor onto; plot (-2, 7), (-1/2, 0), (0, -1), (1/2, 0), (2, 7)



d. $f(x) = x^3 - 5$ is both one to one and onto; plot (-8, -13), (-1, -6), (0, -5), (1, -4), (2, 3)



Sources

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www.purplemath.com/modules/fcns.htm