The Chase

By Lorena Lopez

I'm an endangered species, and I'm called the Philippine eagle. I'm also known as the country's national bird.

Here I am, waiting for my next meal.

Oh, there goes a squirrel nearby. That would be quite easy for me to catch. It has a speed of about 12 mph, so I can easily capture it, even if I have a 40 mph speed. I'll let it play for a while.

But, let's say there is another squirrel 56 miles from where I'm perched right now. How long before I reach it and how far will I fly?

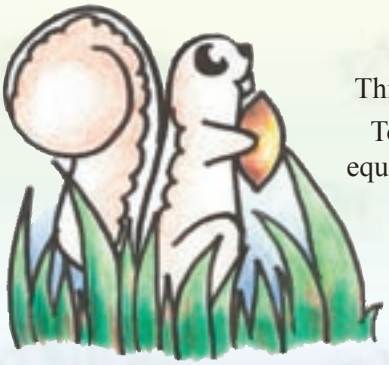
Eagle -------- 56 miles --------squirrel ------- catch point y?

Let's have the catch point distance at y. That means in $40x$ I can reach it, where x is the time I spend in flight. With a 56-mile lead though, the squirrel reaches the point at $12x + 56$ (same duration, but our starting point is different).

$$40x = y$$
$$12x + 56 = y$$

We can solve for x by replacing y in the second equation:

$$12x + 56 = 40x$$
$$12x - 12x + 56 = 40x - 12x$$
$$\frac{56}{28} = \frac{28}{28}x$$
$$2 = x$$



This means that in 2 hours, I have something to eat.

To compute the distance y that I would have traveled, put x value to the first equation:

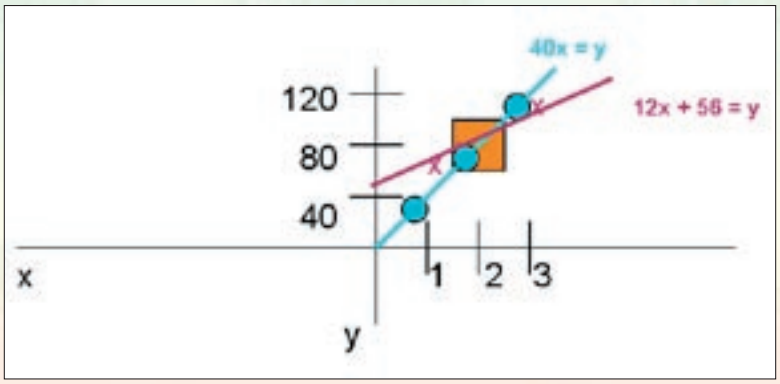
$$40x = 40 \bullet 2 = 80$$

You can't imagine the squirrel running 2 straight hours..

Now let's see how the graph looks like… Let's plot some points for our first equation, (0, 0), (1, 40), (2, 80) and (3, 120); then for the second (0, 56), (1, 68), (2, 80), (3, 92):

Solving Equations with Two Unknowns

Graphing the equations, the point of intersection is the solution to simultaneous equations.



Do you see that point there where the two lines meet? It's an ordered pair (2, 80). And that is the solution to our equations.

All right, nice to meet you, and glad to help with your math. I have to go focus on my food now, and feed my young.

What's that? Will I still be here tomorrow? Who knows? If I don't die of hunger, accident, or old age, I could be caught by some selfish hunters, or disappear quietly, like the many forest friends I've lost, with the only home I know being destroyed day by day by mindless men and greedy developers.

WORKSHEET

1. Feeding time. (Not taking into account factors like the eagle's flight angle.)

   (a) A rodent running at a speed of 3.5 meters per second is 15 meters away from the eagle flying in the same direction at a speed of 15 m/s. Will the rodent survive if it is only 2 meters away from safety?

   (b) Slithering its way deep into the forest at the rate of 28 feet per second, the unsuspecting snake is grabbed by the eagle from the opposite direction. If the snake traveled at least 92 feet, and the eagle 48 feet, after how many seconds will they meet if they traveled the same distance?

   (c) The squirrel runs at a speed of 525 centimeters per second when the eagle spots it. After a 3-second delay, the eagle chases after it at thrice its speed. If the initial distance is 7 875 cm, how long will it take the eagle to catch it?

2. More intersections. Try to solve the following by graphing.

   a. $2x = y$; $3x + 1 = 2y$

   b. $-x + 5 = 3y$; $x + 4y = 7$

   c. $2x = 5 - 2y$; $3 - 4x - y = 0$

   d. $11x - 23 = 10y + 62$; $7y + 5x - 4 = 0$

   e. $6x + 12 = 3y$; $-8x + 8y = 0$

Answer Key:

1. Solution:

a. Eagle ----- 15 m ----- rodent ----- 2 m ----- safety

$3.5x + 15 = y$: rodent (where x is speed, y is distance covered)

$15x = y$: eagle

$15x = 3.5x + 15$

$15x - 3.5x = 3.5x + 15 - 3.5x$

$11.5x = 15$

$11\frac{1}{2}x = 15$

$\frac{23}{2}x \cdot 2 = 15 \cdot 2$

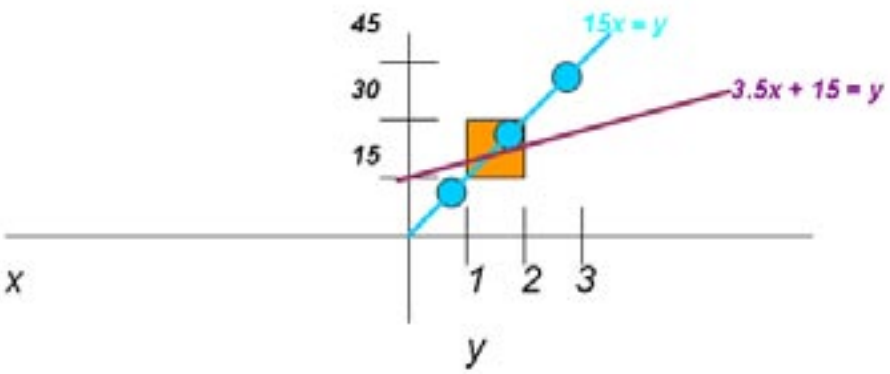
$\frac{23}{23}x = \frac{30}{23}$

$x = 1\frac{7}{23}$ or approximately 1.3m (this means the rodent, which is 2m away from safety, will not make it)

$y = 3.5x + 15 = 3.5 * 1.3 + 15 = 4.55 + 15 = 19.55$ – distance covered by rodent in 1.3 seconds

$y = 15x = 15 * 1.3 = 19.5$ – distance covered by eagle in 1.3 seconds

To see the graph intersecting at point (1.3, 19.5), plot $3.5x + 15 = y$ with points like (0, 15), (1, 18.5)… And $15x = y$ with points like (0, 0), (1, 15)…



b. Eagle ----- y ----- catch point ----- y ----- snake

$32x + 48 = y$

$-(28x + 92 = y)$

$4x - 44 = 0$

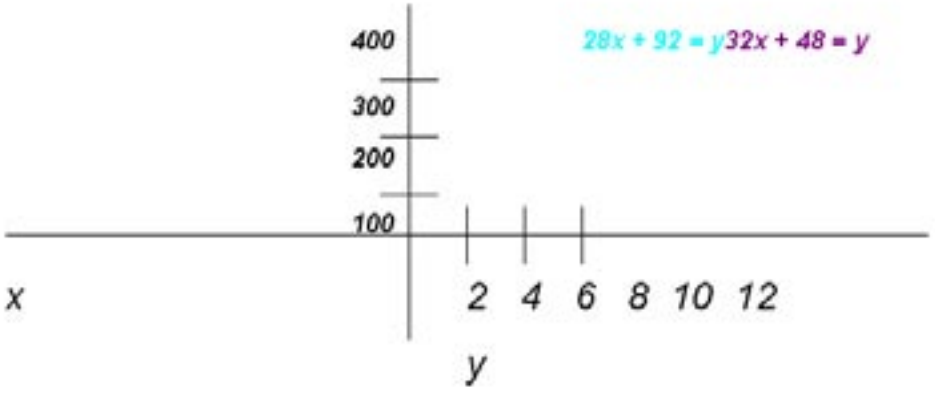
$4x - 44 + 44 = 0 + 44$

$4x = 44$

$x = 11$ seconds until they both meet

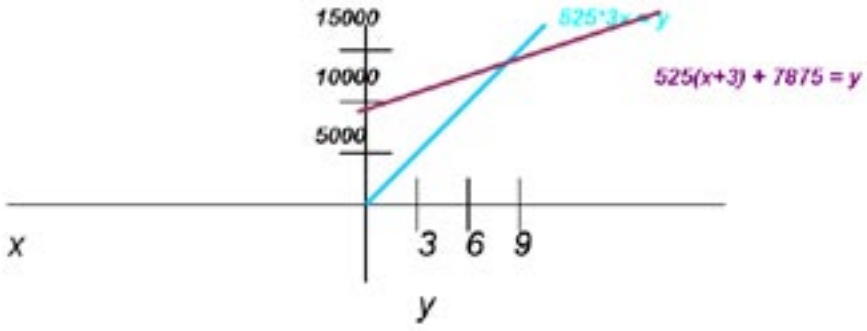
$y = 32 \cdot 11 + 48 = 352 + 48 = 400$ feet traveled by both (which is equal to $28 \cdot 11 + 92 = 308 + 92$)

To see the graph intersecting at point (400, 11), plot $32x + 48 = y$ with points like (0, 48), (1, 80), (2, 112)… and $28x + 92 = y$ with points like (0, 92), (1, 120), (2, 148)…



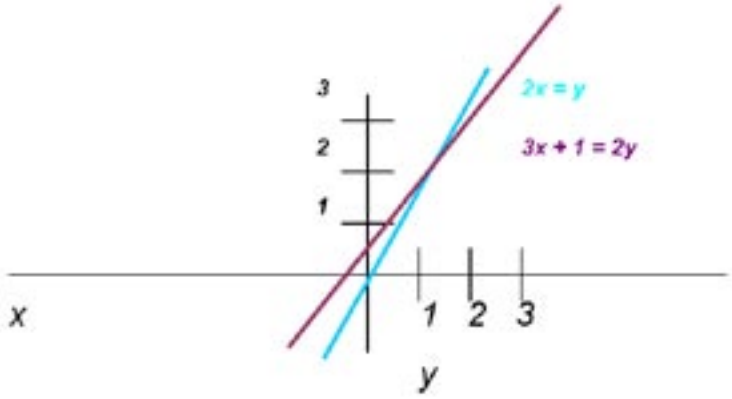
c. Eagle ----- 7,875 m ----- 3-second delay ----- squirrel ----- catch point
The 3-second delay has the squirrel at 1,575 cm head start
$525(x + 3) + 7875 = y$: squirrel (where x is speed, y is distance covered)
$(525 \cdot 3)x = y$: eagle
$525x + 1575 + 7875 = 1575x$
$1050x = 9450$
$x = 9$: time elapsed for the eagle to catch the squirrel
$(525 \cdot 3)(9) = 14175$: distance covered by eagle
To check : $525(9 + 3) + 7875 = 14175$

To see the graph intersecting at point (9, 14175), plot $525(x + 3) + 7875 = y$ with points like (0, 9450), (1, 9975)… And $(525 \cdot 3)x = y$ with points like (0, 0), (1, 1575)…

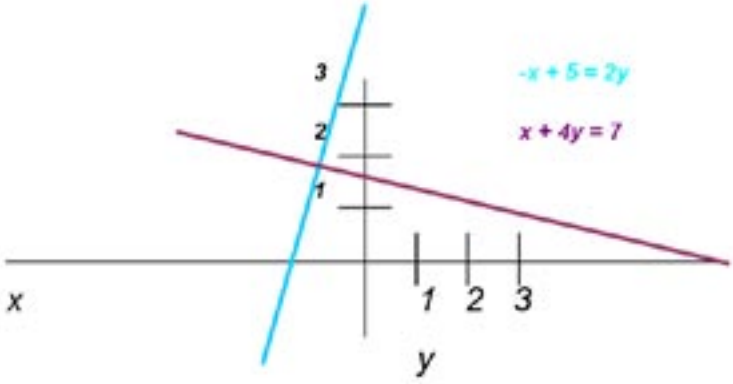


2. Solution:

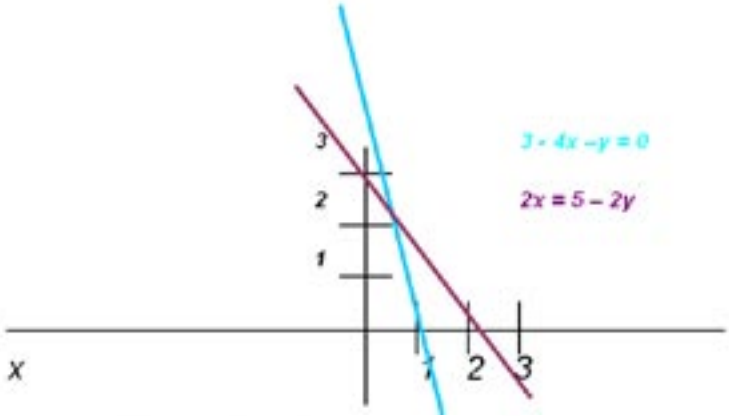
a. Plot $2x = y$ with points like (-1, -2), (0, 0), (1, 2)… And $3x + 1 = 2y$ with points like $(-\frac{1}{3}, 0)$, $(0, \frac{1}{2})$ … [intersect at (1, 2)]



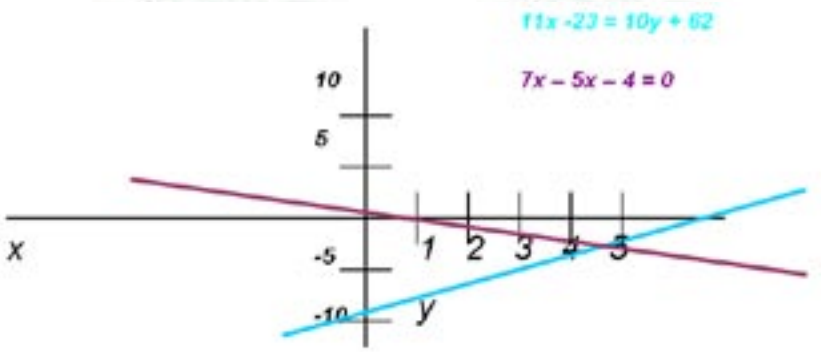
b. Plot $-x + 5 = 3y$ with points like $(0, \frac{5}{3})$, (5, 0)… And $x + 4y = 7$ with points like $(0, \frac{7}{4})$, (7, 0)… [intersect at (-1, 2)]



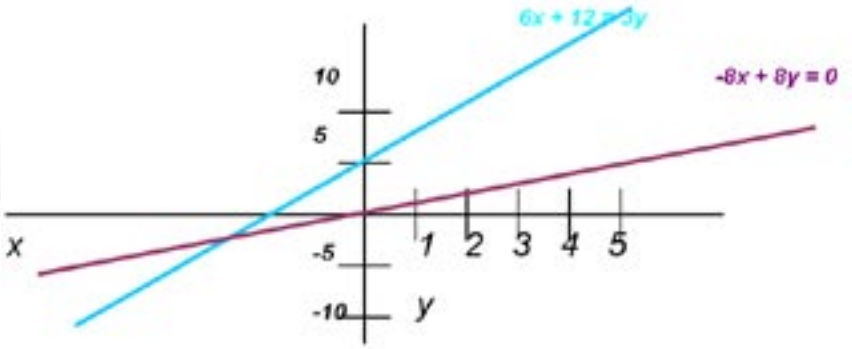
c. Plot $2x = 5 - 2y$ with points like $(0, \frac{5}{2})$, $(\frac{5}{2}, 0)$… And $3 - 4x - y = 0$ with points like (0, 3), $(\frac{3}{4}, 0)$… [intersect at $(\frac{1}{6}, \frac{7}{3})$]



d. Plot $11x - 23 = 10y + 62$ with points like (0, -8.5), $(7\frac{7}{11}, 0)$… And $7y + 5x - 4 = 0$ with points like $(0, \frac{4}{7})$, $(\frac{4}{5}, 0)$… [intersect at (5, -3)]



e. Plot $6x + 12 = 3y$ with points like (0, 4), (-2, 0)… And $-8x + 8y = 0$ with points like (-1, -1), (0, 0), (1, 1)… [intersect at (-4, -4)]



Sources:
http://algebraforathletes.com/html/6_5.htm
www.teach-at-home.com/fastfacts/animalkingdom/Speed.asp
http://universalfacts.blogspot.com/2006/08/maximum-speeds-of-animals.html
www.sunstar.com.ph/static/dav/2006/01/18/feat/philippine.eagle.davao.s.pride.html
http://ngm.nationalgeographic.com/ngm/2008-02/philippine-eagles/mel-white-text.html
www.themathpage.com/alg/simultaneous-equations.htm
www.factmonster.com/ipka/A0004737.html