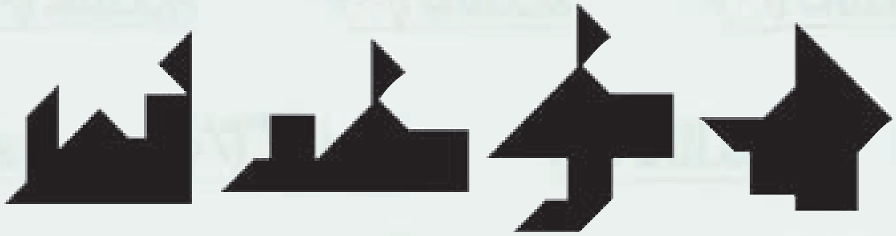
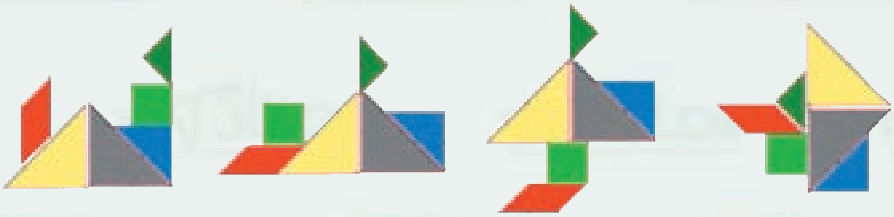
TANGRAM TRICKS

By Ana Marie R. Nobleza

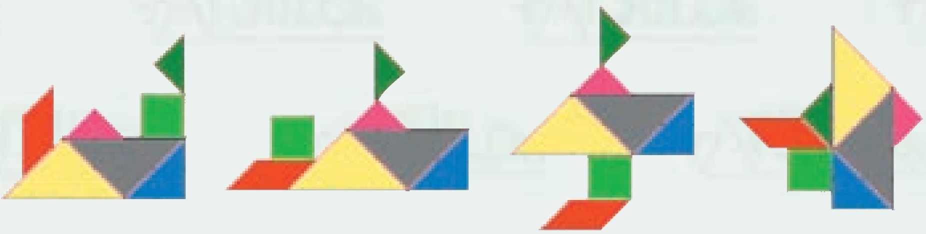
Can you form the figures below with your tangram sets?



At first glance, you may say that these are solutions to the puzzles above.



But should they not be. Why? Count the number of pieces used in forming the supposed solutions. They are only six. But in fact, the tangrams set has seven pieces! What should the real solutions be? The figures below are the real solutions to the puzzles.

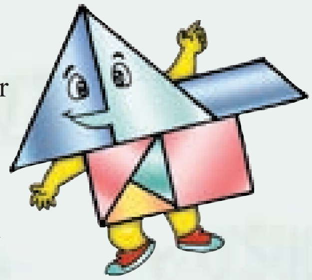


The puzzles that we solved above are only few of the many tangram puzzles that exist at present. Though the existence of tangrams is unrecorded in history, this puzzle has captured the interest of many math puzzlers and even those who are not so mathematically inclined. This is probably because the puzzle seems easy, you only have to flip, rotate and fit the pieces together to solve the figures given.

Rules of the Tangrams

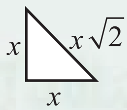
1. You must use all seven tans (that's what the pieces of the tangrams are called).
2. The tans must lay flat, they must touch and none may overlap.

Note: You are free to change these rules depending on how you want to run the puzzle.

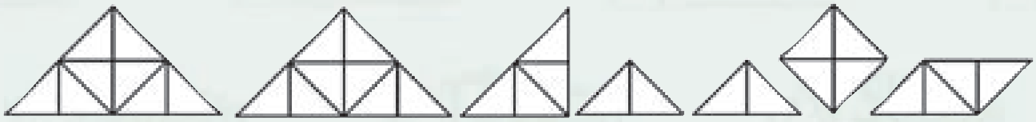


But let's go beyond just flipping and rotating, and let's explore some other clever tricks that these tangram pieces have.

From the article "Tantalizing Tangrams" in the first issue of *Tatsulok* (print edition, school year 2007-2008), we all learned how these tangrams are created. We knew that all seven tangram pieces consist of half squares with this shape:



There are 32 half squares or 16 squares altogether and if $x = 1$ unit, each of these squares must be 1 square unit so the tangram pieces are 16 square units altogether.



Following the classic rules we can build infinitely many nonconvex figures, like those in the opening paragraph, all with 16 square unit areas. But building non-convex figures pose not much challenge than making convex polygons from the tans and studying their perimeters, right?

BUILDING CONVEX POLYGONS WITH THE TANS

Convex Figures

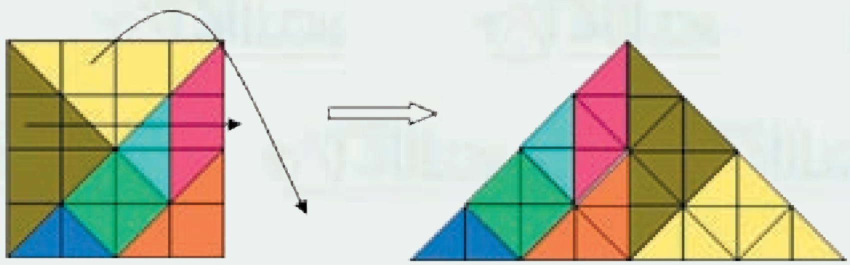
If you choose any two points inside the figure and the whole set of points (line segment) between the points lie inside the figure, then that figure is convex.

Now, let us try to find out how many convex polygons can be built using all of the tans, applying the classic rules. Let us also find out which of these convex polygons will give the least perimeter.

We use the tans with grids so we can right away see if there are inconsistencies in the figures that we are building.

TRIANGLES

Can we build a triangle? Yes, we can! Since the tangrams were created using the square on the left below, building a triangle could be as easy as sliding the two biggest triangles to the right of the remaining figures.

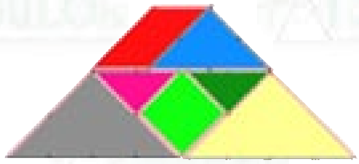


Can you find the perimeter of the triangle, if the tangrams have 16 square unit area altogether?

Did you get $(8 + 8\sqrt{2})$ or approximately 19.31 units? Then you are correct!

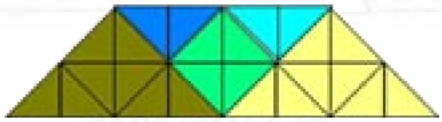
QUADRILATERALS

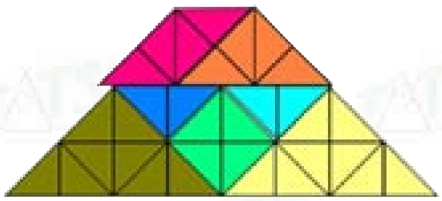
Building a square is the easiest of these puzzles since the tans originated from squares. How about building a rectangle? I will leave as your exercise all of the quadrilaterals that we can build. Find their perimeters too and discover for yourself which among them has the least perimeter. You have to take note though, that this “seemingly” trapezoidal figure below is not a solution to the quadrilateral puzzles.



Why is that so? Using the tans with grids, we can see that the lower part of the figure (composed of the two biggest triangles, square and the two smallest triangles) is already a trapezoid, with upper base four units.

4 units





The inconsistency lies on the upper part (the medium sized triangle and the parallelogram). The two also makes a trapezoid with lower base $3\sqrt{2}$ or approximately 4.24 units, which makes it not fit with the upper base of the trapezoid above. Thus, when put together, the fgures do not exactly make a trapezoid. But there is one trapezoid among the convex quadrilateral puzzle figures and you have to find that out.

OTHER CONVEX POLYGONS

In 1942, two Chinese puzzlers and mathematicians, Fu Traing Wang and Chuan-Chih Hsiung, proved that there were only 13 convex polygons that could be built out of the tans. Of these, one was a triangle, six were quadrilaterals, two were pentagons, and four were hexagons.

Can you build all of these polygons?

Activity

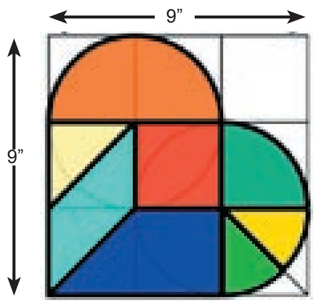
Variants of the Tangrams

Through the years, the tangrams have attracted many puzzlers and that many variants of these have risen. Two of these variants are the ones you will make. The patterns are given below.

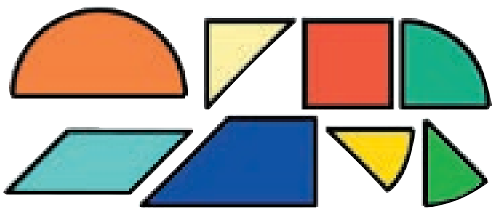
After creating your own sets of these tangrams, create figure puzzles and let your classmates solve them.

THE BROKEN HEART

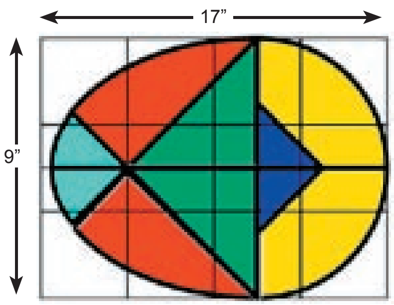
1. Measure a 9-inch-by-9-inch square board and then cut it.
2. Draw a three-by-three grid (distance between lines should be 3 inches) on the board using light lead pencil (see lighter lines in the figure below).
3. Draw the two overlapping circles as shown in the figure . Copy the thick black lines as shown in the figure at the right.
4. Then cut through these lines to produce the eight tans (tangram pieces).



cut along the thick black lines



THE MAGIC EGG



cut along the thick black lines

1. Measure a 12-inch-by-9-inch board and then cut it.
2. Draw a four-by-three grid (distance between lines should be 3 inches) on the board using light lead pencil (see lighter lines in the figure at the left).
3. Copy the black lines as shown in the figure.
4. Then cut through these lines to produce the ten tans (tangram pieces).

Answer: Figure puzzles will vary.

WORKSHEET:

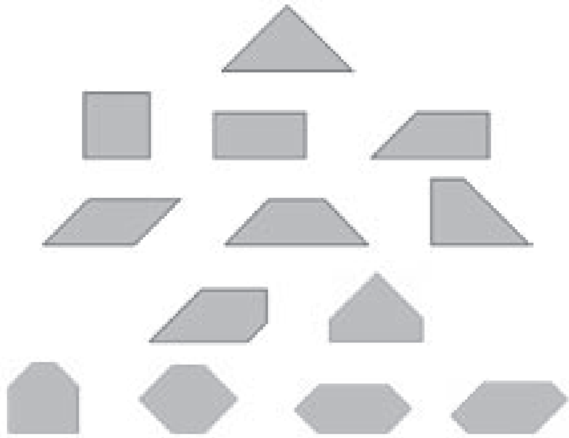
A. The following figures are the convex quadrilaterals that can be created using all of the tans. Figure out how they can be formed and find which of them has the least perimeter.

Triangle

Quadrilateral

Pentagons

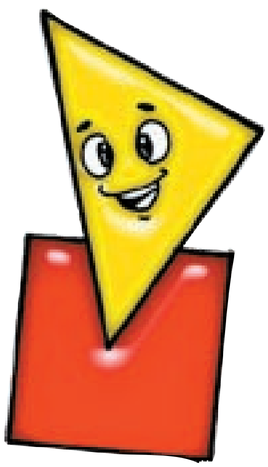
Hexagons



B. Fill up the table below with the perimeters of each convex polygons:

Quadrilateral	Perimeter	Pentagon	Perimeter	Hexagon	Perimeter
1		1		1	
2		2		2	
3				3	
4				4	
5					
6					

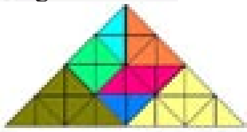
1. Which of the convex polygons has the least perimeter?
2. Which has the greatest?



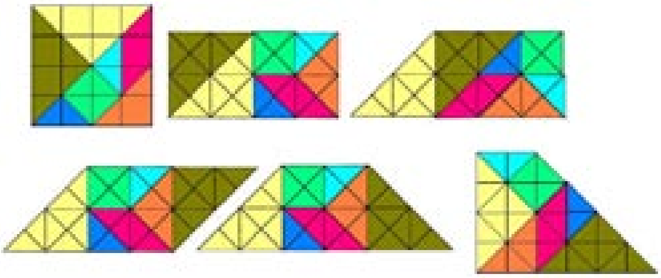
Answers:

A. There may be other ways of forming these figures.

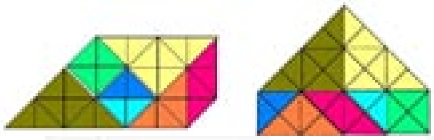
Triangle



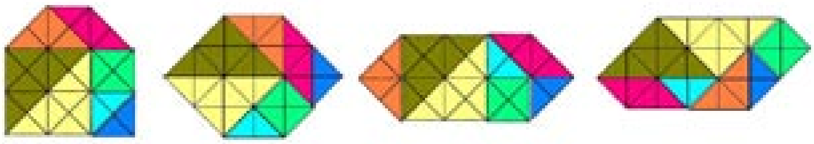
Quadrilaterals



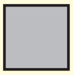
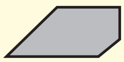
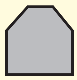
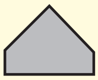
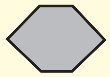
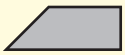
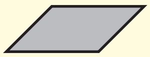
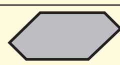
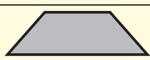
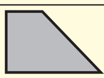
Pentagons



Hexagons



B.

Quadrilateral	Perimeter	Pentagon	Perimeter	Hexagon	Perimeter
	16 units		$12 + 4\sqrt{2}$		$4 + 8\sqrt{2}$
	$12\sqrt{2}$		$12 + 4\sqrt{2}$		$4 + 8\sqrt{2}$
	$4 + 10\sqrt{2}$				$8 + 6\sqrt{2}$
	$8 + 8\sqrt{2}$				$8 + 6\sqrt{2}$
	$8 + 8\sqrt{2}$				
	$12 + 4\sqrt{2}$				

1. The first two hexagons have the least perimeters
2. The triangle, parallelogram, and the isosceles trapezoid have the greatest perimeters.